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RIST

Information Processing at the Digital Age Journal

Volume 26, No 1 (2021)

Special Issue of

The 2nd International Conference on Complex Systems and their Applications

An Efficient Particle Swarm Optimization of RFM for ALSAT2 Images Ortho-rectification

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Abstract

Image ortho-rectification is a standard process in remote sensing for correcting the geometric distortions and relief displacement errors introduced by the payload system during the imaging time. It requires a precise rigorous sensor model or rational function models which are refined using well-distributed ground control points. The Rational function model (RFM) is commonly used because of its simplest model and does not need sensor parameters. Therefore, the RFM terms or also rational polynomial coefficients (RPCs) have no physical significance but depends on many ground control points (GCPs) that make the model prone to the over parameterization problem. The application of meta-heuristic algorithms is suited for RFM optimization. This paper proposes a binary particle swarm optimization BPSO to surmount the issue of over-parameterization and find the optimum combination of RPCs for the RFM by adding a new transfer function. The algorithm is applied to the ALSAT2 images and the results showed the effectiveness and the accuracy of BPSO over the traditional binary literature methods. Furthermore, a hybrid optimization technique is introduced that blends the BPSO concept by adding the genetic operations such as crossover and mutation in order to increase the convergence speed and avoid the local optimum phenomenon. The proposed method gives a better result than the suggested one.

Keywords: High-resolution satellite images; image ortho-rectification; rational function models; particle swarm optimization; hybrid algorithm.

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1. Introduction

One of the most important sources of geographic information systems (GIS) is high-resolution satellite imagery. The high-resolution satellite images are actually used in several contexts at both the industrial and scientific domain, Belfiore and Parente, 2016 but raw images usually contain some significant geometrical distortions. These distortions depend on the device (airplane or satellite), type of sensor and the overall field of view which cannot use the raw images directly for map products in GIS. Ortho-rectification is a standard process for correcting the geometric distortions and relief displacement errors introduced by the payload system during the imaging time Toutin, 2004. It consist of transforming an image's central projection into an orthogonal, uniform-scaled view in which the distorting effects of tilt optical projection and terrain relief are removed, Jeff, 2004. Therefore, the high accuracy potential of ortho-rectification depends on the relationship between images and object spaces, Hu et al., 2004. For that, there have been functions and mathematical models developed either through empirical models (such as 2D/3D polynomial or 3D rational functions) or with rigorous (physical) models, Toutin, 2004. The rigorous models mostly based on the collinear-equation are lack generality because they are complex and its imaging model can differ from one sensor type to another. In addition, parameters such as orbital satellite ephemeris, attitudes, and the physical parameters of the sensor should also be provided for geo-positioning based on a rigorous sensor model; but those parameters may not be accessible because they expose satellite and sensor core technology. Protecting certain key parameters IKONOS, Quick Bird and other commercial satellite imagery vendors have adopted empirical models, simpler and more general imaging platform the renowned type and mainly used in empirical models is rational function model (RFM), Jeff, 2004.

There are two methods to solving the RFM named as dependent-terrain and independent-terrain. In the case of the independent terrain, RFM is solved by using the physical sensor model; otherwise in the absence of the sensor parameters, the RFM is solved by using a set of ground control points (GCPs). This solution depending on the number and the distribution of the ground control point on the terrain is known as dependent-terrain method, Tao and Hu, 2001. The independent-terrain approach, RFM necessitates a large number of accurate, well-distributed GCPs which is a time-consuming and costly process. In addition, RFM coefficients or also rational polynomial coefficients (RPC) have no physical significance which makes it difficult to find their best combination, Yavari et al., 2012. To overcome these problems the binary form of meta-heuristic algorithms can be helpful in optimization and determining the optimum RPCs. Recently, many investigations carried out on the employment of the meta-heuristic approaches for RFM optimizations. Genetic algorithms (GAs) and particle swarm optimization (PSO) are the most useful technique in the literature employed to find the optimum number and combination of RFM parameters.

GA-based approaches for RFM optimization were introduced in Jannati et al., 2012; Valadan Zoej et al., 2012. Zoej et al., 2012 used GA to find an optimum RPC by recommending a three-category division of ground points (GPs): Ground check points (GCPs) are used to estimate RPCs; dependent check points (DCPs) are used to estimate cost functions; and independent check points (ICPs) are used to measure the accuracy of the optimum RFM obtained by the method. In Valadan Zoej et al., 2012 a modified version of GA was employed for RFM optimization. Jannati et al., 2012 have suggested using qualified genes in chromosome body to create some genetically modified (transgenic) chromosomes. The conventional PSO and its modified version were introduced in Yavari et al., 2012 and Yavari, et al., 2013 respectively. Yavari has developed the first PSO-based approach presented in the RFM literature, that used binary particles to decide whether or not the RPCs where present in the RFM structure and designed to be more likely to omitting coefficients rather than maintaining them. This binary modified PSO has outperformed GA in terms of computational time and accuracy in Yavari et al., 2012; Yavari, et al., 2013. Thereafter, PSO's employment has been the subject of several research works as in Moghaddam, et al., 2018 and Gholinejad, et al., 2019.

In this context, this work focused on evaluating the performance of an efficient PSO algorithm based on terrain dependent RFM model applied for the Algerian satellite (ALSAT2) images. The paper is organized as follows: The next section presents the theoretical description of RFM, then the concept of binary PSO for RFM optimization is introduced in section 3, the implementation and methodology of the study technique are provided in section 4. The presentation of the results in the section 5 is divided into two parts, in the first one we give the experiment results of BPSO, and in the second part we present our proposed hybrid algorithm. Finally, we give our conclusions in section 6.

2. Rational function model (RFM)

RFM is widely used by for photogrammetric and remote sensing applications because of its easy implementation and not require any knowledge of the sensor parameters. Mathematically, it define the spatial relationship between ground space (X,Y,Z) and image space (r,c) using a ratio polynomial, Tao and Hu, 2001 as follows :

$$r = \frac{P_1(X,Y,Z)}{P_2(X,Y,Z)} \qquad (1)$$

$$c = \frac{P_3(X,Y,Z)}{P_4(X,Y,Z)} \qquad (2)$$

Where:

$$P_i = a_{i,0} + a_{i,1}X + a_{i,2}Y + a_{i,3}Z + a_{i,4}XY + a_{i,5}XZ + a_{i,6}YZ + a_{i,7}X^2 + a_{i,8}Y^2 + a_{i,9}Z^2 + a_{i,10}XYZ + a_{i,11}X^3 + a_{i,12}XY^2 + a_{i,13}XZ^2 + a_{i,14}X^2Y + a_{i,15}X^3 + a_{i,16}YZ^2 + a_{i,17}X^2Z + a_{i,18}Y^2Z + a_{i,19}Z^3 \qquad (3)$$

where a_i, b_i, c_i, d_i indicate the RFM coefficients and are referred to as Rational Polynomial Coefficients RPCs or Rational Function Coefficients, RFCs see Tao and Hu 2001; Chen et al. 2006.

The Unknown RFCs can be solved with two methods. The first one is known as Terrain-independent which utilizes a physical sensor model. Thus, it requires the availability of some information on the sensor (attitude and orbital parameters), which is based on the following steps, see Tao and Hu 2001:

1- Image Grid Determination,

2- Use of the physical sensor model to create a 3D object grid,

3-Use of RFM to solve the RPCs by using the image points and their corresponding ones in the 3D object grid,

4- The RFM Accuracy can be evaluated through comparing the image grid points calculated over RFM with the coordinates of the original image grid points by calculating the difference.

The second method is called Terrain-dependent in which the physical sensor information is not necessary but it is based on the distribution and the number of ground control points GCPs, Tao and Hu, 2001; Chen et al. 2006. For this case, coefficients b_0 and d_0 of the denominator polynomials are considered to be equal to 1 to simplify the calculations. So the number of the remaining unknown RPCs is 78, which requires at minimum 39 GCPs to be solved, see Jannati et al., 2017 using a standard least squares (LS) method according to the following steps:

Linearization and reformulation of nonlinear equations (1) and (2) as follows:

$$P_1(X,Y,Z) - rP_2(X,Y,Z) = 0 \qquad (4)$$

$$P_3(X,Y,Z) - cP_4(X,Y,Z) = 0 \qquad (5)$$

The above equations can then be written as follows, by using n GCPs (Tao and Hu, 2001):

$$y = Ax + e \qquad (6)$$

Where:

A: design matrix

y: observations vector
e: residuals vector
x: vector of RPCs.

The least-squares (LS) method can be applied to determine RPCs as follows :

$$x = (A^T A)^{-1} A^T y \qquad (7)$$

3. The binary particle swarm optimization for RFM optimization

Particle swarm optimization is one of the most common meta-heuristic optimization algorithms inspired by social intelligence and cooperative behavior displayed by various species to fill their needs in the search space. The first version of the particle swarm algorithm is developed by James Kennedy and Russell Eberhart in 1995 which works in continuous search space, see Eberhart and Shi, 2001; Kennedy and Eberhart, 1995.

In RFM optimization, the binary form is applied. The standard binary PSO can be defined by the following equations , Eberhart and Shi, 2001; Kennedy and Eberhart, 1997:

$$v_{ij}(t+1) = w.v_{ij}(t) + c_1 r_1 \left(p_{ij}(t) - x_{ij}(t)\right) + c_2 r_2 \left(p_{gj}(t) - x_{ij}(t)\right) \qquad (8)$$

where:

- t is the iteration number.
- v_{ij} is the velocity of the bit j of ith particle bounded within a range of $[v_{min}, v_{max}]$,
- x_{ij} is the position of the bit j of ith particle.
- Pg denotes the best particle of the swarm, that is the particle with the best objective function value, and the best previous position of the ith particle in its own searching trajectory is recorded and represented as Pi.
- W is the inertia weight.

The update function for the position is defined as follows:

$$x_{ij}(t+1) = \begin{cases} 1, & \text{if} r_{ij} < \emptyset(v_{ij}) \\ 0, & \text{otherwise} \end{cases} \qquad (9)$$

Each element in the vector velocity is regarded as the input of a normalizing function (transfer function) and usually is a sigmoid function, see Gruen and Kocaman, 2008 which determines the probability in the range of [0,1].

The algorithm begins with a population of particles which is a set of RFMs structure generated at the first run randomly in a string of the binary values. This implies that each particle is a combination of one and zero, indicating the presence or absence of the corresponding coefficient RPC in the RFM. In this work, the RFM with 78 parameters was used; hence each particle is represented by a string of 78 binary values as indicated in figure bellow.

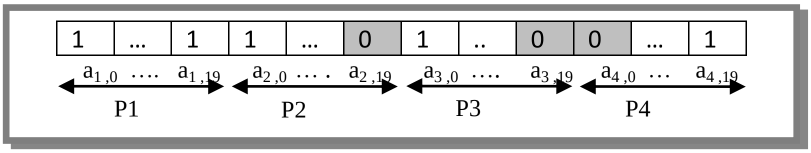

Fig.1. Particle representation

RFM optimization aims to minimize the number of terms and maintains sufficient accuracies therefore the normalizing function should be structured to be more likely to omit, rather than maintain the terms. Hence in our algorithm called BPSO-RFO the *tanh* function is used as the normalizing function due it deliver successful results as it demonstrated in Yavari, et al., 2013, so the bits updating is performed with eq.9 using the velocity of the bit calculated by eq.8, the normalizing function formula is as follows:

$$\emptyset(v_{ij}) = \begin{cases} \tanh(v_{ij}), & \text{if} v_{ij} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

The algorithm is repeatedly updated until a criterion for termination is reached. In this study, we declared the maximum number of iterations (tmax) to be termination condition.

4. Implementation and methodology

In this paper, two high-resolution multispectral images were used of size of 1750×1750 pixel. These images are acquired by the Algerian satellite ALSAT2 over Winterthur city (Switzerland). The first one “Winterthur1” consists of 18 GCPs and the second “Winterthur2” contains 20 GCPs. These GCPs detected directly from terrain when the measurements were realized in August 2007, see Gruen and Kocaman, 2008. Fig.2 shows Winterthur images of the ALSAT2 satellite within their CPs location.

The RFM optimization process is applied under the CPs in three different parts. First part of these points is employed to estimate the unknown coefficients of the model, which is called Ground Control Points (GCPs). The second part of CPs is used to calculate the fitness value for each particle named Dependent Checkpoints (DCPs). And the last part of these points is used just for accuracy assessment that is addressed as independent check points (ICPs). Generally, the most common measure of accuracy used is Root Mean Square Error (RMSE) given by this equation

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\hat{x}_i)^2+(y_i-\hat{y}_i)^2}{N}} \tag{11}$$

Where: N is the total number of points, (x_i,y_i) the estimated coordinate (x, y), $(\hat{x}_i, \hat{y}_i)$ denote the actual coordinate (x, y).

We can summarize our methodology of RFM optimization based on the proposed algorithm as follow:

1- Estimation of the RPCs proposed by each particle by using a set of GCPs and the least square method (LS).

2- Evaluation of each particle by calculating the RMSE over DCPs which is the cost function.

3- Updating the RFM structure according to the BPSO-RFO algorithm. Table 1 depicts the parameters of the proposed method.

The experiments have been implemented using MATLAB on a personal computer with a 2.40GHz Intel Core i3 CPU and an 8 Gb RAM. The following table depicts the different parameters of the BPSO-RFO algorithm.

Table 1.Parameters used in the BPSO-RFO algorithm.

Population size		30
v	v_{max}	+3
	v_{min}	-3
w		0.7
t_{max}		200
C1		1.5
C2		1.5

(a) *(b)*

Fig.2. Dataset images: (a) Winterthur1; (b) Winterthur2.

5. Results and discussions

The experiments have been divided into two sections: The first section consist to evaluating the performance of our BPSO-RFO algorithm in terms of accuracy, stability and convergence speed. In the second section, we propose the hybrid HPSO-RFO algorithm in order to enhance the results.

5.1. First section: Performance of BPSO-RFO

To evaluate the performance of the proposed algorithm BPSO-RFO, different testing experiments are carried out by selecting different combinations of well distributed GCP/ICP. In all experiments a 20% of the GCPs were selected randomly as DCP to calculate the cost function of each particle. In such issues, the RMSE over DCPs was widely used as a cost function. The accuracy of the obtained results is determined by two measuring criteria: first the RMSE calculated over ICPs which is indicator of the accuracy, second the standard deviation (STD) which is a proper indicator of stability.

Furthermore, a comparison was conducted with the conventional binary PSO and standard GA with the same combinations of GCPs/ICPs. The genetic operators crossover and mutation probabilities used in the experiment are respectively 0.75 and 0.001. The binary tournament and two-point crossover methods were employed. As the meta-heuristic algorithms given a different result in each execution, each algorithm is executed 10 times. The particle with the lowest cost function was selected as the best one and mentioned in the following table.

Table 2. The accuracy and stability results of different algorithms

Data sets	GCP/ICP	RMSE over ICP (the best run among 10 runs)			STD of RMSE over ICPs (10 runs)		
		BPSO-RFO	Conventional PSO	Conventional GA	BPSO-RFO	Conventional PSO	Conventional GA
Winterthur1	15/3	0.8726	0.9836	**0.8287**	**0.2435**	0.3304	1.6440
	12/6	**0.8839**	1.1383	2.9475	**1.0130**	1.4216	204.4333
	8/10	**1.6471**	205.3063	133.8651	**0.4785**	9.4025	2.7009e+04
Winterthur2	15/5	**0.8484**	0.8844	0.8781	**0.0822**	0.2578	2.9202
	12/8	**0.8827**	1.4710	0.8976	**1.0938**	2.9363	500.7724
	08/12	**1.6258**	1.0265e+03	742.5804	**231.2884**	2.9080e+03	711.0490

As shown in Table above, the BPSO-RFO achieves clearly better results than the conventional PSO and GA in most cases in term of accuracy and in all cases in term of stability. The value of RMSE over ICP shows the high accuracy of the proposed methods which can optimize the RFMs coefficients and obtain a sub-pixel on accuracy just with 12 GCPs for the two images. If the number of GCPs is less than 12 the accuracy is degraded in the worst case to 1.64 pixels which is appropriate for photogrammetric and remote sensing applications. Unlike the conventional PSO and GA, the accuracy values have degraded to 1000 and 700 pixels respectively, this due to the local minima problem. In term of stability, our algorithm shows high stability especially with 12 and 15 GCPs. In the overall view of the results our method is more stable than both conventional PSO and GA, this due to the transfer function (eq.10) which is designed to be more omit of the RPC than preserve them, this helped the BPSO-RFO to minimize the number of RPC with acceptable accuracy.

5.2. Second section: The proposed hybrid algorithm for RFM optimization

When comparing conventional GA to the conventional PSO, it is clear that GA is more accurate and stable; this is due to the genetic operations such as mutation and crossover which give to GA more diversity in the search space and guide research towards the best solution. Moreover and from the results obtained in the previous section, BPSO-RFO and GA give better results and are approximately equal compared to conventional PSO method. In this section, we propose to study a hybrid technique named HPSO-RFO that blends the concept of BPSO and GA by integrating the genetic operations mutation and crossover to BPSO algorithm.

An intelligent stochastic crossover procedure is used in the proposed HPSO-RFO to protect the essence of particle swarm optimization in the binary domain (eq.12).

$$x_{ij}\begin{cases} x_{ij} & if \quad 0 \le r_{ij} \le \alpha \\ P_{ij} & if \quad \alpha < r_{ij} \le 2\alpha \\ Pg_{ij} & if \quad 2\alpha < r_{ij} \le 1 \end{cases} \qquad (12)$$

In eq.12,α denotes the crossover probability that has been predetermined. In this experiment, α= 33%, which means that each bit x_{ij}, in a particle has a 33 % probability of remaining in its state as determined by equations (9,10) and a 66% chance of being replaced by a bit from its previous best and current global best particle. Another popular genetic algorithm (GA) operator for maintaining population variety is mutation. The mutation operator is used in HPSO to accomplish two purposes. First, it may keep the swarm diverse; second, it can make escape easier to any local optimum. The mutation operation is applying in HPSO-RFO by generate a random number in the interval [0-1] and compared to mutation probability m which equal 0.02 is this research in order to performing mutation operation as shown in eq.13

$$x_{ij} = \begin{cases} not(x_{ij}), & if r_{ij} \le m \\ x_{ij}, & \text{otherwise} \end{cases} \qquad (13)$$

The flowchart of the proposed hybrid algorithm HPSO-RFO is illustrated in figure 3 and the different results obtained with the proposed hybrid algorithm are depicted in the table below.

Table 3. The results of the HPSO-RFO algorithm.

Data sets	GCP/ICP	Optimum (the best run among 10 runs)			Among 10 runs	
		Number of RPC P1,P2,P3,P4	RMSE of ICPs	RMSE of GCPs	Average RMSE of ICPS	STD RMSE of ICPs
Winterthur1	15/3	9 ,1, 6, 2	**0.4257**	0.3971	1.0625	0.8737
	12/6	5 ,1, 8,3	1.5157	0.2159	2.7408	1.0941
	8/10	5 ,1,4, 2	**1.2268**	0.2200	8.5279	7.9802
Winterthur2	15/5	4,1,9, 4	**0.8295**	0.3357	1.0583	0.1161
	12/8	4,3, 4,3	**0.8564**	0.2053	1.1799	0.3039
	8/12	3 ,2,3 ,1	**1.5179**	0.1719	88.9673	242.9930

Table 3, shows the obtained optimum RMSEs of ICPs and GCPS as well as the number of the best found RPCs [P1,P2,P3,P4] by the proposed hybrid algorithm HPSO-RFO applied for the Winterthur data set. From the experimental results obtained, we approve that our proposed hybrid algorithm HPSO-RFO has a better accuracy compared to BPSO and GA in the majority of cases this is due to the mutation and crossover added in which give it more diversity and variety in solution than other literature tested methods. The HPSO-RFO algorithm gives good results and achieves an optimum structure of RFM (minimum RPCs) with accuracy less than 1.51 in all cases compared to other tested literature methods. Therefore, the HPSO-RFO outperforms the initial proposed approach (BPSO-RFO) that was presented in section 5.1, due to the two additional operations crossover and mutation, which boost the convergence speed and prevent the local optimum phenomena, the obtained results of the optimum run (table 3) have proved the superiority of HPSO-RFO and reveal a 5.7% improvement over BPSO-RFO.

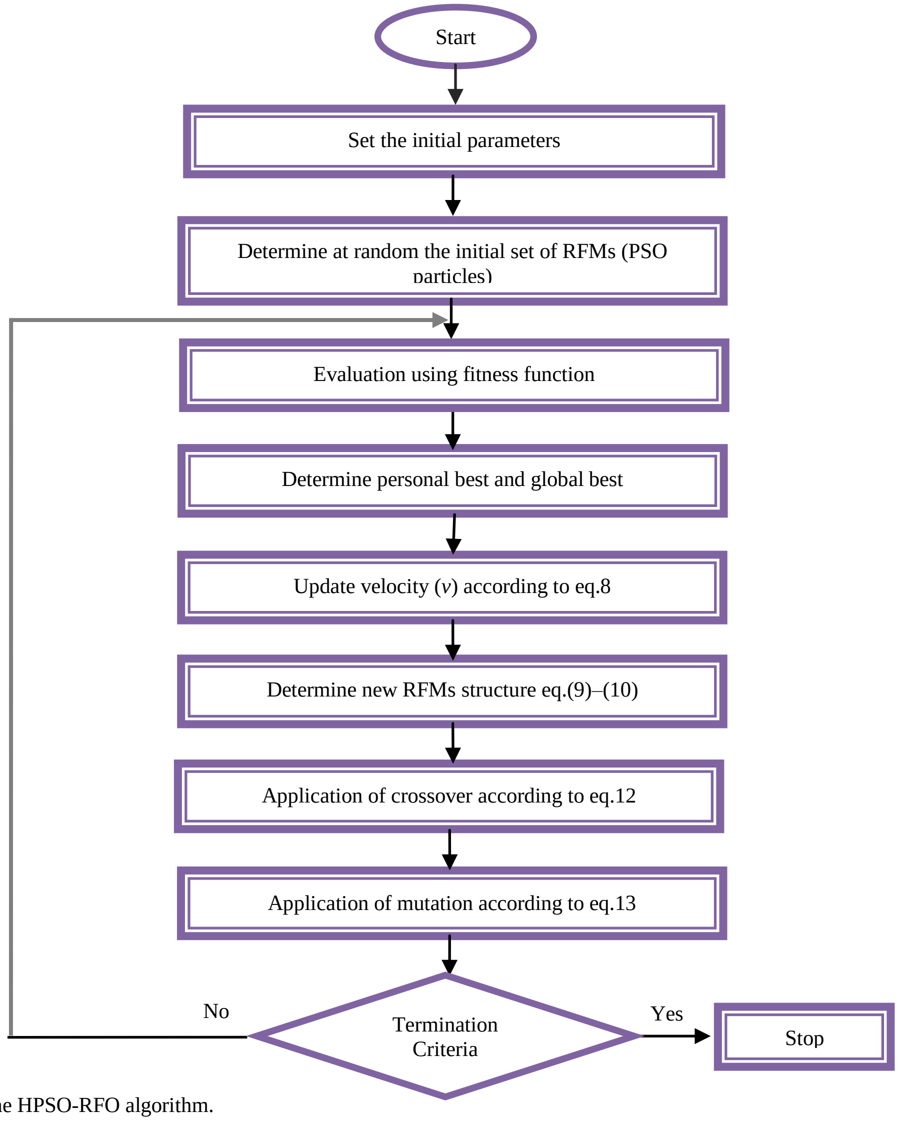

Fig.3. The flowchart of the HPSO-RFO algorithm.

The two last columns of table 3 indicate the stability and the average accuracy of the proposed algorithm over the 10 runs. The stability of the algorithm was positively reflected by the accuracy in which the average RMSE of ICPs achieves an acceptable accuracy when implementing 15 and 12 GCPs for the two data sets. A value of 2.7 pixels is obtained in the worst case. However, the stability of the algorithm is degraded for the case of 8 GCPs. Accordingly, to the results of the most cases, the HPSO-RFO is more suitable and provides higher accuracy and good stability.

To test the convergence speed of the tested methods, the best run among the 10 runs is selected in this section for convergence speed analysis. Fig.4 demonstrates the convergence curve of the literature methods with a very limited number of GCPs (GCP=8) for Winterthur1 data set. As observed in figure, the conventional PSO and GA have a slow convergence than BPSO-RFO and HPSO-RFO which are much faster.

Our proposed algorithms shows significant performances compared to BPSO-RFO due to the transfer function and the mutation and crossover operations in HPSO-RFO that does not just improve the accuracy but also the convergence speed.

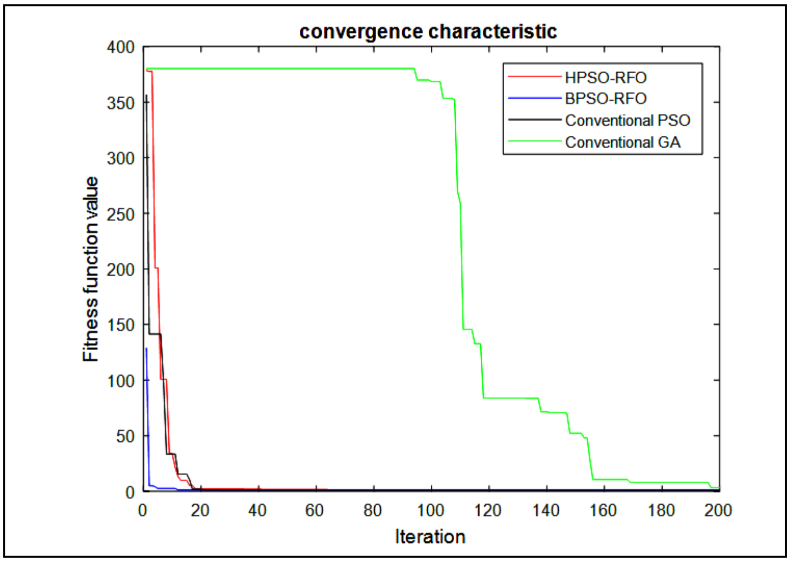

Fig.4. Convergence speed of the tested methods.

6. Conclusion

This paper presents a modified binary PSO of RFM optimization for ALSAT2 images ortho-rectification. The proposed binary PSO can achieve a sub-pixel on accuracy with a very limited number of GCPs. The results obtained demonstrate the performance of our proposed algorithms BPSO-RFO in term of accuracy and stability comparing to the conventional binary PSO and conventional binary GA in the most cases. In term of convergence speed the proposed binary PSO confirm its superiority to other tested method, the BPSO-RFO converges rapidly less than 15 iterations.

In the second part of this paper, a HPSO-RFO algorithm is proposed by mixing PSO and GA concepts to increase the accuracy of ortho-rectification by presenting the optimum RFM coefficients with better accuracy than the other tested methods this is due to the genetic operations (mutation and crossover) added,. Moreover, the algorithm has a fast convergence compared to conventional PSO and GA (less than 20 iterations) to converge towards the optimum results.

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